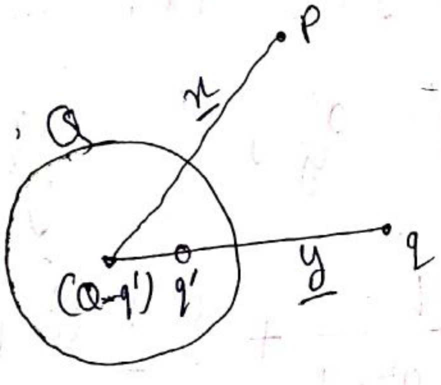
Point charge in presence of charged insulating Sphere:- In previous prob. we have seen that a point charge q near a grounded sphere induces surface charge density of total amount $q' = -\frac{aq}{y}$ and is distributed over surface in such a way as to be in equilibrium under all forces acting. If we wish to consider prob. of an insulating conducting sphere with total charge Q in the presence of a point charge q; we can build up the Soln for the potential by linear superposition.



We can imagine grounded conducting sphere. with its induced charge q' distributed over its surface. We then dis-connect the ground wire and add to sphere an amount of charge (Q- q') this brings total charge on sphere upto Q. To find potential we note that added charge (Q-q') will distribute itself uniformly over surface since, electrostatic forces due to point charge q are already balanced by q'. Hence Potential due to added charge (Q-q') will be same as if a point charge of that magnitude were at origin at least for points outside of sphere.

Potential is superposition of previous case and potential of a point charge (Q-q') at origin:

$$\phi(x) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{|\underline{x}-\underline{y}|} + \frac{q'}{|\underline{x}-\underline{y}'|} + \frac{Q-q'}{|x|}\right]$$

but, $y' = \frac{a^2}{y}$, $q' = -\frac{aq}{y}$

then,

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{|\underline{x}-\underline{y}|} - \frac{aq}{y\left|\underline{x} - \frac{a^2}{y^2}\underline{y}\right|} + \frac{Q + \frac{aq}{y}}{|x|}\right]$$

Force acting on charge q can be written directly from Coulomb's law. It is directed along to direction of q and will have the magnitude ——— (8)

$$\underline{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{y^2}\left[Q - \frac{a^3 q(2y^2-a^2)}{y(y^2-a^2)^2}\right]\frac{\underline{y}}{y}$$

——— (9)

In the limit of $y \gg a$

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for, $y >> a$

force reduces to usual Coulomb's law for two small charge bodies. But close to sphere the force is modified bcoz induced charge distribution on surface of sphere.

eqn (9)

for various value of Q/q –

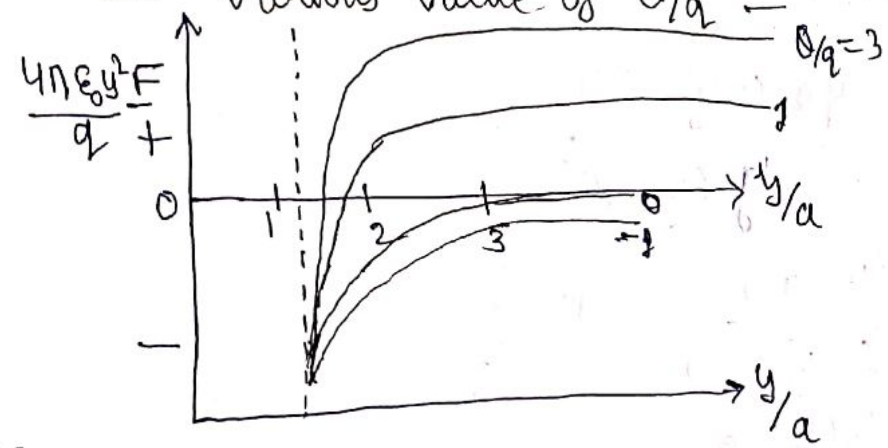

fig:– asymptotic dependence of force has been divided by. $\frac{4\pi\epsilon_0 y^2 E}{q^2}$ versus $\frac{y}{a}$ for $\frac{Q}{q} = -1, 0, 1, 3$ plotted. the force on point charge q due to an insulated conductiong sphere of radius a having total charge Q. Positive values mean a repulsion, negative an attraction.

Force is always attractive at close distances bcoz of induced surface charge.

Force is expressed in units of $\frac{q^2}{4\pi\epsilon_0 y^2}$ if sphere is charged with oppositely q or is uncharged. force is attractive at all distances even if Q is of same

Sign as q. forces become ~~att~~ attractive at very close distances. In the limit Q>q, point of zero force unstable equlibrium point is very close to sphere namely at

$$y \simeq a\left(1+\frac{1}{2}\sqrt{\frac{q}{Q}}\right)$$

in ~~limit~~ find this, by

$$Q - \frac{qa^3(2y^2-a^2)}{y(y^2-a^2)^2} = 0$$

$$\frac{y(y^2-a^2)}{(2y^2-a^2)} = a^3 \cdot \frac{q}{Q}$$

$$\frac{y(y^2-a^2)}{(2a^2-a^2)} = a^3 \cdot \frac{q}{Q}$$

$$\frac{a(y^2-a^2)}{a^2} = a^3 \cdot \frac{q}{Q}$$

$$(y^2-a^2)^2 = a^4 \frac{q}{Q}$$

~~$y = a^2 + a^4 \frac{q}{Q}$~~

$$y^2 - a^2 = a^2 \frac{q}{Q}$$

$$y^2 = a^2 + a^2 \frac{q}{Q}$$

$$y \simeq a\left(1+\frac{1}{2}\sqrt{\frac{q}{Q}}\right)$$

asymptotic value of force is attained as soon as charge q. more than a few radii.

This ex. gives generaly property which explains why an excess charge on sphere does not immediately leaves surface bcoz of mutual repulsion. of individual charges.

As soon as, an element of charge is remove from surface imaged force tends to attractive back. If sufficient work is done charge can be remove from surface to ∞. The work function of a matter is in larged part just work done against attractive imaged force to remove an electron from surface